

Complex Analysis

Ques - Show that when  $0 < |z| < 4$

$$\frac{1}{4z - z^2} = \sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n+1}}$$

Sol<sup>n</sup> When  $|z| < 4$ , we have

$$\begin{aligned} \frac{1}{4z - z^2} &= \frac{1}{4z \left(1 - \frac{z}{4}\right)} \\ &= \frac{1}{4z} \left(1 - \frac{z}{4}\right)^{-1} \\ &= \frac{1}{4z} \left[1 + \frac{z}{4} + \left(\frac{z}{4}\right)^2 + \left(\frac{z}{4}\right)^3 + \dots\right] \\ &= \frac{1}{4z} + \frac{1}{4^2} + \frac{z}{4^3} + \frac{z^2}{4^4} + \dots \end{aligned}$$

ie

$$\frac{1}{4z - z^2} = \frac{1}{4z} + \frac{1}{4^2} + \frac{z}{4^3} + \frac{z^2}{4^4} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n+1}}$$

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